November 13, 2017 MATHEMATICS 218 NAME Key

Time : 60 minutes QUIZ 2 ID# ------------------

Fall 2017-18

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12
2 W	1 W	11 W	2 M	1 M	11 M	4 M	3 M	2 Th	11 F	4 F	5 F

PROBLEM GRADE

PART I

1 ------- / 16

2 ------- / 16

3 ------- / 9

4 ------- / 7

PART II

5	6	7	8	9	10	11	12	13
a	a	a	a	a	a	a	a	a
b	b	b	b	b	b	b	b	b
c	c	c	c	c	c	c	c	c
d	d	d	d	d	d	d	d	d
e	e	e	e	e	e	e	e	e

5-13 ------- / 28

PART III

14	15	16	17	18	19
T	T	T	T	T	T
F	F	F	F	F	F

14-19 ------- / 24

TOTAL ------- / 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 0 & -1 & -2 \\ 2 & 4 & 6 \end{pmatrix}$

(a) Find a basis for the null space N(A) of A. (Do not prove it is a basis)

[8 points]

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 3 & 4 & 0 \\ 0 & -1 & -2 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \xrightarrow[-2R_1+R_4]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right]$$

Pivot

$$\xrightarrow[-2R_2+R_3]{1R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{row-echelon form}$$

$x + y + z = 0$ x & y are Leading variables
$y + 2z = 0$ $z = t$ is a free variable

So $z = t \Rightarrow y = -2z = -2t$
$\Rightarrow x = -y - z = 2t - t = t$

Null space of $A = N(A)$
$= \left\{ \begin{pmatrix} t \\ -2t \\ t \end{pmatrix} \middle| \ t \text{ any real number} \right\}$

Basis of $N(A) = \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\}$, $\begin{pmatrix} t \\ -2t \\ t \end{pmatrix} = t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

(b) Find a basis of the column space Col(A) of A. (Do not prove it is a basis)

[8 points]

Columns 1 & 2 in row echelon form contain Pivots

So Columns 1 & 2 in The original matrix form a basis for Col(A)

So Basis of $Col(A) = \left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ -1 \\ 4 \end{pmatrix} \right\}$

2. Prove or **disprove** the following statements:

(a) Let W={A∈ M_{2x2} | A is a symmetric non-invertible matrix}. Then, W is a <u>subspace</u> of M_{2x2} .

[8 points]

FALSE

Example $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} \in W$ & $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \in W$

But $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}}_{\text{invertible}} \notin W$

(b) Let U={A∈ M_{3x3} | A is a skew-symmetric ($A^t = -A$) matrix}. Then, U is a <u>subspace</u> of M_{3x3} of <u>dimension</u> 3.

[8 points]

TRUE

* $U \neq \emptyset$ since $0 \in U$

* $A, B \in U \Rightarrow A^t = -A$, $B^t = -B$

$\Rightarrow (A+B)^t = A^t + B^t = -A - B$

* $A \in U \Rightarrow (cA)^t = cA^t = -cA \Rightarrow cA \in U$

So U is a subspace of $M_{2\times 2}$

$A \in U \Rightarrow A = \begin{pmatrix}0 & t & s\\-t & 0 & r\\-s & -r & 0\end{pmatrix}$ $A^t = -A$ (skew-symmetric)

So $\dim U = 3$

3. Let V be a vector space and **u**, **v**, **w**, are vectors in V such that Span{**u**, **u**+**v**}= Span{ 2**u**, 3**v**, 3**u**+2**w**}. Show that **w** is a linear combination of **u** and **v**.

[9 points]

So $(3u+2w) \in \text{Span}\{u, u+v\}$

$\Rightarrow 3u+2w = c_1 u + c_2(u+v)$

$\Rightarrow 2w = (c_1+c_2-3)u + c_2 v$

$\Rightarrow w = \frac{1}{2}(c_1+c_2-3)u + \frac{1}{2}c_2 v$

So w is a Linear combination of u and v

4. Let $T: V \to V$ be a linear operator on a vector space V such that $T(T(v)) = T(v)$ *for all* $v \in V$. Prove that every vector $v \in V$ can be written as a sum $v = u + w$, *where* $u \in N(T)$, *and* $w \in R(T)$

[Hint: Find $T(T(v)-v)$]

[7 points]

$\forall v \in V, \ T(T(v)) = T(v)$

$$\Rightarrow T(T(v)-v) = T(T(v)) - T(v)$$
$$= T(v) - T(v) = 0$$

So $T(v) - v \in N(T)$

So $u = T(v) - v \in N(T)$

$\Rightarrow v = \underset{N(T)}{\underset{\uparrow}{-u}} + \underset{R(T)}{\underset{\uparrow}{T(v)}}$ = sum of a vector in $N(T)$ and a vector in $R(T)$

PART II. Circle the correct answer for each of the following problems (Problem 5 to Problem 11) <u>IN THE TABLE OF THE FRONT PAGE</u>. [4 points for each correct answer (No penalty)].

5. Let A= $\begin{pmatrix} 1 & 2 & 1 & 6 & 3 & 8 \\ 2 & 4 & 0 & 2 & 2 & 2 \\ 0 & 0 & 1 & 5 & 2 & 7 \end{pmatrix}$. Then

$\xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & 2 & 1 & 6 & 3 & 8 \\ 0 & 0 & -2 & -10 & -4 & -14 \\ 0 & 0 & 1 & 5 & 2 & 7 \end{bmatrix}$

$\xrightarrow{\frac{1}{2}R_2+R_3} \begin{bmatrix} 1 & 2 & 1 & 6 & 3 & 8 \\ 0 & 0 & -2 & -10 & -4 & -14 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(a) Rank (A)=2, Nullity (A)=4
(b) Rank (A)=4, Nullity (A)=2
(c) Rank (A)=3, Nullity (A)=3
(d) Rank (A)=1, Nullity (A)=5
(e) Rank (A)=5, Nullity (A)=1

[4 points] So Rank (A)=2

6. Let $T: P_3 \to R^2$ *be the linear transformation defined by*:

$T(p(x)) = \begin{pmatrix} p(-1) \\ p(0) \end{pmatrix}$. Then dim N(T)=

$p(x) = ax^3 + bx^2 + cx + d \in N(T)$

$T(p(x)) = \begin{pmatrix} -a+b-c+d \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(a) 0
(b) 1
(c) 2
(d) 3
(e) None of the above

$d = 0$, $c = -a+b$

[4 points]

7. Let $W=\{p(x) \in P_2 \mid p(1)+p(-1)=0\}$ .Then

$P(x) = ax^2 + bx + c$
$P(1) + P(-1) = 0$
$= a+b+c+a-b+c$
$= 2a+2c = 0$
$\Rightarrow c = -a$
$P(x) = ax^2 + bx - a$
$= a(x^2-1) + bx$

(a) B={ x^2+1} is a basis for W
(b) B={ - x^2+1} is a basis for W
(c) B={ - x^2+1, x} is a basis for W
(d) B={ x^2+1, x} is a basis for W
(e) none of the above

[4 points]

8. If T: $R^2 \to R$ is a linear transformation such that $T\begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1$, then

$\dim N(T) + \underbrace{\dim R(T)}_{\overset{\shortparallel}{1}} = 2$

(a) N(T) = {0}.
(b) R(T) = R .
(c) Dim (N(T))=2.
(d) N(T) = R^2.
(e) none of the above.

[4 points]

9. Let A be a 4x4 matrix and let R be a row-echelon form of A. Which of the following is **FALSE**?

(a) dim (row (A)) = dim (col (A))
(b) N(A) = N(R)
(c) row (A) = row (R)
(d) col (A) = col (R)

[4 points]

10. If A is any 3x4 matrix, which of the following statements is **FALSE**?

(a) The columns of A are linearly dependent.
(b) Row (A) is a subspace of R^4.
(c) The rows of the matrix form a basis for row (A).
(d) The system AX = 0 has infinitely many solutions.

[4 points]

11. Let U be the subset of P_3 given by

$$U=\{p(x) = ax^3 + bx^2 + cx \ + d \in P_3 \mid p(0) = 0 \, and \; p''(x) = 0\}.$$

Then dim U=
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4.

$p(0) = 0 \Rightarrow d = 0$

$p'(x) = 3ax^2 + 2bx + c$

$p''(x) = 6ax + 2b = 0 \Rightarrow a = 0 ; b = 0$

So $p(x) = cx$

[4 points]

<u>PART III</u>. Answer **TRUE** or **FALSE** only (Question 12 to 19) **<u>IN THE TABLE OF THE FRONT PAGE</u> [3 points for each correct answer, <u>-1 point penalty for each wrong answer</u>]**

F 12. If U and W are subspaces of a vector space V with dim U = dim W, then U = W.

T 13. The polynomials $2+3x$, $3-4x^3$, $x+x^3$, $1+x-x^2$, $5x+2x^2$ are linearly dependent in P_3

F 14. Let W= $\left\{\begin{pmatrix} a+b & a+b \\ 2a+2b & -a-b \end{pmatrix} \in M_{2\times 2} \mid \; a,b \in R\right\}$, then W is a <u>subspace</u> of $M_{2\times 2}$ of dimension 2.

T 15. If $T:V\to W$ is a linear transformation and if $\{T(\mathbf{u}_1), T(\mathbf{u}_2), T(\mathbf{u}_3)\}$ is a linearly independent subset of W, then $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is linearly independent in V.

F 16. Let V= $\left\{\begin{pmatrix} x \\ y \\ x^2 \end{pmatrix} \in R^3 \mid \; x, y \in R\right\}$. Then V is a subspace of R^3.

T 17. The set of polynomials $\{1+x, x^2+3, x^2+x, 5\}$ is a spanning set for P_2.

T 18. If A is a 3x3 matrix such that $A^2 = 2I$ then the rows of the matrix A form a basis for row (A). A is invertibl

F 19. If A is a 4x6 matrix with rank A = 3, then dim N(A)= 1

[24 points]